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Reg. No. : .....

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Sixth Semester B.Sc. Degree Examination, April 2018
First Degree Programme Under CBCSS
Mathematics
Core Course X
MM 1642 : LINEAR ALGEBRA
(2014 Admn. Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **first 10** questions are **compulsory**. **Each** question carries **1** mark.

1. For which choice of x will the following pairs be linearly dependent ?

   a) $\begin{pmatrix} x \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

   b) $\begin{pmatrix} x \\ x^2 \end{pmatrix}, \begin{pmatrix} 1 \\ x \end{pmatrix}$

2. Show that the parametric equation $X = V + t(U - V)$ represents the line through U and V if U and V are any two vectors which are not equal.

3. Find the angle between the vectors $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$.

4. Find the distance from the $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ to the line $y = 2x$.

5. Let S be the transformation which assigns to each vector X, the reflection of in the line along the vector $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$. If $X = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$, find S(X).

6. Let A be the transformation with matrix $\begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix}$. Find the image of the along the vector $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ under A.

7. Define eigenvalue and eigenvector of a linear transformation.

8. Define Isometry transformation.

9. Classify the quadric surface $4x^2 - 9y^2 + z^2 + 36 = 0$.

10. Find a non-zero vector $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ in R^3 which is orthogonal to each of the vectors $\begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

SECTION – II

Answer **any 8** questions from this Section. **Each** question carries **2** marks.

11. Find the characteristic polynomial of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 0 & 4 \\ 6 & 4 & 5 \end{pmatrix}$.

12. Show that the transformation with matrix $\begin{pmatrix} 2 & 0 & 0 \\ 4 & -1 & 0 \\ 2 & 3 & -1 \end{pmatrix}$ is invertible.

13. Let P denote the projection on the line along U. Find m(P).

14. Let T be a length preserving transformation and the matrix of T is $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

    Show that T is a rotation when the determinant is 1 and a reflection when the determinant is –1.

15. Prove that $R_{\frac{\pi}{2}}$ has no eigenvalue.

16. In terms of coordinates of $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, calculate the images of $R^1_{-\frac{\pi}{2}}$ and R^3_{π}.

17. If $m(T) = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 3 & 1 \\ 3 & 1 & 4 \end{pmatrix}$ and $m(S) = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 3 & 1 \end{pmatrix}$, then show that $m(ST) \neq m(TS)$.

18. Define Shear and Permutation matrices.

19. Let S and T be linear transformation of R^3 such that ST = I. Prove that TS = I.

20. Find all eigenvalues of the matrix $\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$.

21. Find an equation in new coordinates of the form $\lambda_1 u^2 + \lambda_2 v^2 + \lambda_3 w^2 = 1$ for the quadric surface $-4x^2 + 2y^2 + 3yz + 2z^2 = 1$.

22. If T is an isometry of R^3, then prove that T has 1 or – 1 as an eigenvalue.

SECTION – III

Answer **any 6** questions from this Section. **Each** question carries **4** marks.

23. Let A be a linear transformation and let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be its matrix. If $|ad - bc| > 0$, then show that A preserves the orientation.

24. If $\vec{A} \times \vec{B} = 0$ then prove that $\vec{A}$ and $\vec{B}$ are linearly dependent.

25. Let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be an arbitrary matrix. Prove that there exists elementary matrices e_1, e_2, e_3, e_4 and diagonal matrix $\begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix}$ such that
$e_1 e_2 \begin{pmatrix} a & b \\ c & d \end{pmatrix} e_3 e_4 = \begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix}$.

26. Find the inverse of $R_{\frac{3\pi}{2}}$.

27. Find all solution of the non-homogeneous system
$x + 2y - 4z = -4$
$2x + 5y - 9z = -10$
$3x - 2y + 3z = 11$.

28. Prove that an isometry of R^3 preserves the dot product.

29. Let m be a symmetric 3×3 matrix and let M be the corresponding linear transformation. If X_1 and X_2 are the eigenvectors corresponding to the distinct eigenvalues t_1 and t_2 then show that $X_1 . X_2 = 0$.

30. Find a non-zero vector $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ in R^3, which is orthogonal to each of the vectors $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$.

31. Classify the quadric surface
$x^2 + 3y^2 + z^2 + 2xy + 2xz + 2yz - 2x + 4y + 2z + 12 = 0$.

SECTION – IV

Answer **any 2** questions from this Section. **Each** question carries **15** marks.

32. Let $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$. Find all eigenvalues and corresponding eigenvectors. Also find the matrices P and D such that $P^{-1}AP = D$ is diagonal.

33. State and prove Spectral theorem in R^3.

34. a) Let T be a linear transformation of R^3 which satisfies the condition : $T(X) = 0$ implies that $X = 0$. Prove that T has an inverse.

b) Solve the system
$x_1 - 2x_2 + 3x_3 = 0$
$x_1 + x_2 + x_4 = 0$
$4x_3 - x_4 = 0$

c) Show that $\left(e_{12}^{s}\right)^{-1} = e_{12}^{-s}$.

35. a) Let T be an orientation preserving linear transformation of R^3. If Π is any parallelepiped then prove that
volume $(T(\Pi)) = |\det(m(T))|(\text{volume}(\Pi))$.

b) Given three points in the plane $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ which are not collinea show that there exists a circle which passes through the three points.